Homework 10

#1 Let X be a space

(a) Prove: X is locally path-connected $\Leftrightarrow$ the path components of every open set are open

(b) Prove: If X is locally path connected, then the connected components of X = the path-connected component of X

#2 Let (C,∂) and (C',∂') be chain complexes. Define graded groups $(C\otimes C')_n = \bigoplus_{i+j=n} (C_i \otimes C'_j)$ and homomorphisms $d: C_p \otimes C'_q \to (C\otimes C')_{p+q-1}$ by $d(x\otimes y) = \partial x \otimes y + (-1)^p x \otimes \partial' y$. This defines $d: (C\otimes C')_n \to (C\otimes C')_{n-1}$. Prove that $\{C\otimes C', d\}$ is a chain complex. Define a homomorphism

$$\alpha: H_p(C) \otimes H_q(C') \to H_{p+q}(C\otimes C').$$

(Note: This problem is for those who know about tensor products.)

#3 Define $T: I \to S^1$ by $T(t) = (\cos 2\pi t, \sin 2\pi t)$. $T \in Q_1(S^1)$ $\partial T = 0$ Then $T' = T + D_1(S^1) \in Z_1(S^1)$ (singular theory) $[T'] \in H_1(S^1) \approx \mathbb{Z}$. Without using the fundamental group show that $[T']$ is a generator of $\mathbb{Z}$. (Hint: $H_1(S^1) \approx \tilde{H}_0(S^0)$.) Can you find a generator of $H_n(S^n) = \mathbb{Z}$?

#4 Massey, p.210, problem 5.2. Express the homology of $S^p \vee S^q$ in terms of the homology of S^p and S^q. Similarly for $S^p \vee S^q \vee S^r$.

* #5 Use #4 to show that $S^1 \vee S^1 \vee S^2$ and $S^1 \times S^1$ (the torus) have the same homology. Show that they do not have the same homotopy type.

* #6 Massey, problem 6-8, p. 217. Assume the following $A, B \subseteq \mathbb{R}^n$ $h: A \to B$ a homeo. Then $h(A^o) = B^o$ and $h(\dot{A}) = \dot{B}$.

#7 Massey, problem 6-9, p. 217.

#8 Massey, problem 6-10, p. 217.